Reversing the Academic Trend for Rural Students: The Case of Michelle Opbroek

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This case study explores the interactions between a teacher, her students, and a culturally based math curriculum in a fifth and sixth grade classroom in rural Alaska. The case attempts to identify and illuminate factors that created a rich learning environment while implementing *Star Navigation: Explorations into Angles and Measurement,* a module from the series, *Math in a Cultural Context* (MCC). This case describes how the teacher facilitated the embedded Yup'ik cultural knowledge into lively, mathematical communication and learning made relevant to a non-Yup'ik group of students. Students' pre- and post-test results showed strong gain scores as well as high absolute post-test scores, placing this class in the small category where a rural treatment group outperformed all urban treatment and control groups. Thus, this compelling case provides an example of a classroom and curricular learning environment that reverses national trends for rural students in general and shows potential for Alaska Native students in particular. Further, it provides examples of factors that other teachers, administrators, and teacher educators can employ in their own teaching and classes to create more effective math classrooms.

Introduction

The academic gap between American Indian and Alaska Native (AI/AN) students (Institute for Government Research, 1928) and their Caucasian peers as well as the rural-urban divide (Johnson & Strange, 2005) have been well documented. Alaska Native majority, rural school districts continue to typically score between the 10th and 20th percentile on reading, language, and math on Alaska's state benchmark exam, while urban schools fall between the 40th and 70th percentile (Alaska Department of Education & Early Development, 2005). Less well documented and of primary importance to the field are cases that document the reversal of these trends for both rural (Silver, 2003) and AI/AN populations (Demmert & Towner, 2003).

In this paper we examine how a Caucasian teacher in a mixed Athabaskan and Caucasian community has successfully implemented *Math in a Cultural Context* (MCC). This case describes how the teacher, Michelle Opbroek, facilitates the embedded Yup'ik cultural knowledge into lively, mathematical communication and learning made relevant to this non-Yup'ik group of students. Further, the case describes the mathematical topics that the students were struggling to understand (angles, distance, etc.) and the class dynamics that allowed them to negotiate their understanding as a whole group.

Originally this lesson was identified as a possible case example because we observed the enthusiastic engagement by the students around mathematics and mathematical communication and argumentation. The case became more convincing once the test scores were compiled: Students' pre- and post-test results showed strong gain scores as well as high absolute post-test scores (see Figure 1). See the introduction to this special edition for further results from the statistical analysis.

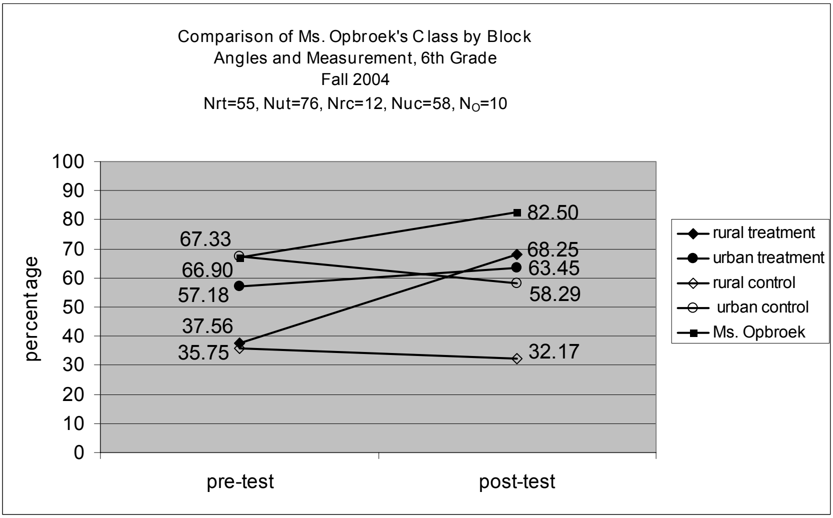

Figure 1. Student test results comparing Ms. Opbroek's class to the four blocks.

These results place this class in the small category where a rural treatment group has outperformed all urban treatment and control groups. Thus, this compelling case provides an example of a classroom and curricular learning environment that reverses national trends for rural students in general and shows potential for Alaska Native students in particular. There are other persuasive aspects of this case that address additional national concerns, such as identifying the classroom processes that (a) encourage math communication and argumentation, (b) create effective classroom use of problem solving, and

(c) allow for student-generated questions, conjectures, and proofs (National Council of Teachers of Mathematics, 2000). Lastly, this case raises an interesting question regarding culturally based curriculum developed from one culture and used in another: How does the teacher use and adapt cultural components of MCC to develop meaningful math understanding for students of another cultural group?

Case Overview

This case example discusses lessons observed in Ms. Opbroek's classroom while she used an MCC module titled *Star Navigation: Explorations into Angles and Measurement,* appropriate for the sixth grade. The math of the module was created from knowledge shared by Frederick George, an accomplished navigator and Yup'ik elder from Akiachak, Alaska. Frederick has worked tirelessly with our project for over 10 years, developing this module as well as others. The *Star Navigation* module uses Frederick's methods of measuring between objects at a distance using hand measures, knowledge of his surroundings, and his self-learned patterns in the movement of shadows and the stars as a basis for understanding angles and measurement. As one author who also co-penned this curriculum, Frederick's knowledge was fascinating but difficult to translate into math curricula. Imagine being on the frozen, seemingly undifferentiated Alaska tundra in the middle of the night with only the stars to guide your way. Frederick does this year after year in all kinds of weather, using the embedded mathematical knowledge he learned from his elders. It was this knowledge that generated the module. Despite our enthusiasm, we were uncertain if students would find Frederick's Yup'ik cultural knowledge compelling enough to want to learn the embedded mathematics.

This case builds directly on Frederick George's everyday use of mathematics, in which he makes explicit measuring angles and relative distance used in navigating during the day and at night. As the students progressed through the module, they discussed a variety of mathematical topics concerning angles. In the lesson used for this case example, the mathematical discussion moves along nicely for the first 10 minutes: Students seem engaged and many have an opportunity to share. However, the main question stemming from one student's response guided the remaining hour of student-led mathematical argumentation. The discussion contained conjectures and proofs and students used a variety of ways that they were comfortable with to explain their answers. Although the teacher played an integral part in ensuring that the students were on task, respectful to their peers, and progressing towards an end, her role became that of a facilitator.

All transcripts in this case are extracted from a lesson videotaped on November 18, 2004. However, this lesson was not an anomaly. For two years, one author has been observing Ms. Opbroek's classroom as she uses both MCC modules and other curricula. Furthermore, the co-author also observed this same classroom with previous teachers and did not see the same type of classroom enthusiasm. The examples provided within this case are just a small piece of what was observed throughout the entire school year.

Background to the Case

Nenana is located in interior Alaska at the confluence of the Nenana and Tanana rivers, 55 road miles southwest of Fairbanks on the main road system connecting the cities of Fairbanks and Anchorage. It has a population of 549 (Alaska Department of Commerce, 2005) and includes a mixture of Athabaskan Indians and non-Natives. Unlike many other rural Alaska villages not on a road system, which have only air access on a gravel landing strip, Nenana is quite accessible by air, river, road, and railroad.

The student population at the Nenana City Public Schools (NCPS) is composed of 50% Athabaskan, 49% Caucasian, and the remaining 1% of African Americans and Asians. As of 2003, about 55% of students receive free or reduced lunches, compared to the state average of about 33% (National Center for Education Statistics, 2005).

Over 40% of the year-round jobs are government-funded, including the city, tribe, Nenana School District, Yukon-Koyukuk School District, and the Alaska Department of Transportation highway maintenance. Many of the students in this case have families that rely on subsistence foods (i.e., salmon, moose, waterfowl, and berries) and consistently participate in outdoor activities such as hunting, fishing, trapping, hiking, and camping.

At the time of this lesson, Ms. Opbroek was in her second year of teaching at NCPS in Nenana. Previously she taught four years in Bethel, Alaska, a rural Yup'ik regional hub, and one year in the continental United States. Ms. Opbroek brings with her the fundamental philosophy of constructivist teaching. Her degree is in elementary education through junior-high with a science emphasis, and her methods courses all used the constructivist approach. In her own math studies, she has progressed through calculus II. Through previous analysis of project data, we have found that teacher math background correlates positively with student achievement as shown in Table 1 below. When analyzing student gain score based on teacher math background using an ANOVA we found $F(1, 283)=6.086$ and $p=0.014$. This means that teachers who completed higher level math courses had students outperform those in other classes at statistically significant levels for this data set.

Beyond minimal culturally based learning from her work in the classroom, Ms. Opbroek spent two weeks of her first summer in Bethel working with and under the direction of Yup'ik elders, helping their youth create culturally based science fair projects. It was here that she learned about Yup'ik traditional ways, from storytelling to fishing, berry picking, steam bathing, language, and crafts, as well as having the opportunity to observe the interactions and behaviors between youth and elders. Further experience came in her second and third summers where she was (again) a science teacher working with sixth-grade students from around the Lower Kuskokwim at an outdoor school/camp situation

Table 1
Summary of test results based on teacher math background showing mean (M), and standard deviation (SD) in parentheses by teacher degree

Math Background 2nd grade, Fall 2002	N	Pre-test (%)	Post-test (%)	Gain Score (%)
HS	35	36.3 (18.5)	48.4 (19.0)	12.1 (17.8)
BS	231	37.8 (18.5)	54.4 (21.5)	16.6 (18.1)
Minor	19	28.6 (13.6)	54.6 (23.8)	26.0 (24,9)

for a month each time. Although she was not learning culture from the elders, she interacted with the Yup'ik youth throughout their days at camp and learned a lot about their perspectives on culture and their lives in modern times. She was able to use these experiences in her classroom to help her connect with her students. Ms. Opbroek states, "I do not believe I would have been such an effective teacher without these experiences."

The classroom in this case is a fifth- and sixth-grade multiage class with 16 students; 14 of them have been together for most of their schooling. The class has 10 sixth-graders and six fifth-graders. One new sixth-grade student came from another village at the beginning of the school year and the other new sixth-grade student came from a military home just a few days before this lesson. This year only three students are Athabaskan and the rest are Caucasian. Nine of the students are in their second year with this teacher, including one fifth-grade student who was held back. These nine also participated in a different MCC unit the previous year. There are two sets of siblings in the class. Three students have repeated a grade at some point in their elementary career and one was home schooled for at least a year. This multiage student group with familial relations or long-time jointly schooled students is typical in rural Alaska, both on and off of the road system.

The classroom itself is a large space mainly divided into two areas. On one side of the classroom, rows of desks face a white board, establishing a space for teacher-led instruction and a place for individual students to call their own. The other side of the classroom consists of a rectangular table by a chalkboard and three additional round tables. This area is used primarily for small-group work. When necessary, all or part of the classroom can be cleared to create empty floor space.

In the mornings, students learn individual skills and subjects, focusing in the areas of reading, writing, and mathematics. The Saxon Math (Larson, 2004) program is strictly used in the NCPS. During this 100-minute block of time, students spend 45 minutes attending band class with their grade group, and participate in physical education twice a week.

The afternoons are primarily reserved for lab-type activities. Five students are pulled out for Title 1 reading assistance two to three times a week. Once a week, students receive health instruction from the school counselor and have library time. There are no special education students in this group of students.

According to the school, only one out of 16 students in the class this year is considered advanced mathematically. Four of them are considered poor and/or struggling, and the remainder of the students appears to be about average on paper. In 2003 when the sixth-grade students were tested as fourth-graders on the Terra Nova, their standardized test results of 77% placed them above the state average of 65% in math and slightly below the state average in reading, 69% vs. 71% (National Center for Education Statistics, 2005). Rural sites in Alaska can be coded as single-site districts, multiple-site districts, or hub sites. Hub sites are typically larger villages that act as a travel center from the major cities (Anchorage, Fairbanks, and Juneau) to the smaller villages. In Alaska, most rural school districts rank low on standardized tests; however, many of the single-site and hub-site rural school districts tend to score higher. NCPS is a typical single-site district in that their test scores are closer to urban sites than their other rural counterparts.

Star Navigation Module

In the first section of the *Star Navigation* module containing Activities 1-3, students are introduced to Frederick George, navigating in general, gathering observational data including shadow measurements, and experimenting with various ways of measuring at a distance. At the time of this lesson, students had used the module for about 10 days and were still completing Activities 2 and 3.

The class spent time on the snow-covered baseball field, gathering sun and shadow data, observing the environment, and measuring distances between landmarks using hand measures learned from Frederick, such as those on the left in Figure 2.

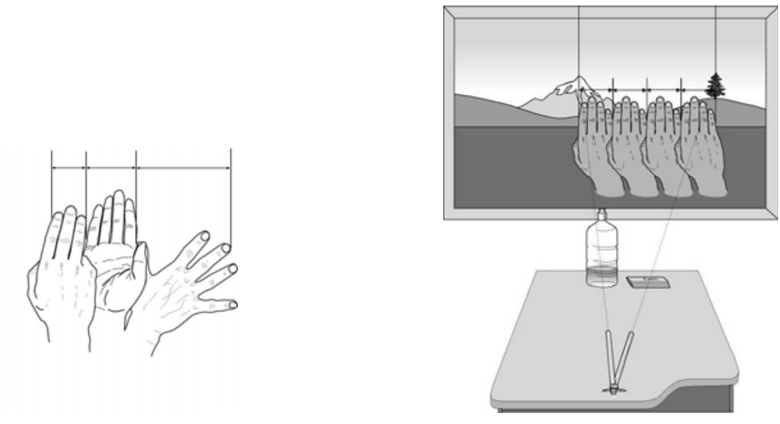

Figure 2. Examples of Frederick George's hand measures used during navigating (left) and their use with the straw angle in the classroom (right).

In the module, a tool is designed to connect the mathematical idea of an angle to Frederick's method of measuring at a distance. The tool, a straw angle, simply consists of connecting two straws together with a brass fastener. The activity then follows: Pick two far-away objects outside the window and place two objects on the desk in the same line of sight as the far-away objects. Note that the outside objects do not have to be an equal distance from your location. Placing the straw angle on the desk, show from your perspective how the objects form the same angle or how the outside objects fall on the same ray paths as the inside objects. Once students have the straw angles, objects on the desk, and the far-away objects in line, ask them if they could place the straw angle in another location and still keep all the objects lined up. Explain that Frederick uses his hand measurements to estimate the distance between the objects and the angle they form in relation to his location. The final result of the activity looks similar to what is shown on the right in Figure 2 (Adams & Kagle, 2005 draft).

Mathematically, there are several ideas that can be investigated. First, if the same hand measure is used for both sets of objects from the same original point, then the hand measures are approximating an angle measurement and the focus is on perspective. Second, if the viewpoint is changed, producing a different perspective, the measurement is changed. Third, if different measures are used for the far-away and nearby objects, then the activity measures the arc length of the angle at different points on the rays, which is not the same as measuring the actual angle or the amount of rotation formed by the rays. Fourth, in the cases when the arc length or the straight line distance between the objects can actually be measured it will provide a measurement that differs from the angle measurement thus possibly causing confusion. Together, these ideas bring to light the issues of what is an angle and what is being measured.

Methodology

As in other cases within this special issue, the research methodology parallels the collaborative work of the overall MCC project. Qualitative data consist of (1) classroom observations, (2) videotapes and transcriptions of those lessons, (3) teacher interviews, and (4) transcriptions of discussions among consultants during video analysis. With this case, we first sought to only explain through the analysis how the teacher enacted the curriculum through specific pedagogical strategies that increased students' mathematical understandings somewhat absent of cultural connections. We did not expect the seemingly Western framework of the classroom to resonate as strongly as it did with the Yup'ik consultants. To our surprise, Ms. Opbroek's class also fit into their Yup'ik framework for a productive classroom in both process and output. This is further explained in the discussion section.

<u>Data Collection</u>

1. Classroom observations were conducted three times while the class was using the *Star Navigation* module. Since one author lives in the same

village, she knows the students and most of their families and had observed in Ms. Opbroek's class during the previous year as well as observing previous fifth- and sixth-grade teachers.

2. Videotape and transcriptions were imported into Transana and analyzed for math communication, teacher questioning, and classroom flow.
3. Video analysis meeting transcriptions were audio taped and transcribed. During a video analysis session in March 2005, 12 consultants viewed the lesson and made comments. Those present included two of the authors, other university researchers, retired school district administrators, and Yup'ik consultants.
4. Teacher interviews were conducted after each lesson to gain insight on Ms. Opbroek's view of the lesson. An additional interview was conducted after the module was completed focusing on specifics to this case study.

Case Analysis

Before the lesson begins, the chairs are in rows and the room is empty. As students arrive back into their classroom, Ms. Opbroek explains that they will be working on their star navigation unit. She asks them to clear the floor so they can start with a discussion reviewing yesterday's activity. The students choose to move their desks out of the way and rearrange their chairs into a circle instead. Ms. Opbroek makes a quiet exclamation that this was not what she expected. The lesson begins with Ms. Opbroek asking students to review the previous activity, relating back to what happened the day before. Alice,[1] a sixth-grade Athabaskan student, volunteers to demonstrate how Frederick George would use his hand measures to measure the distance between two far-away objects, see Figure 3. After Alice shares this, Ms. Opbroek asks the class, "Does anyone want to add anything else?" She is already structuring the class for students to share and allowing students to respond to other students. Collin adds, "It also changes depending on how you move your hands." Kathy, another sixth-grade student, not only extends the developing math discourse but opens a new line of inquiry that the curriculum itself does not address by applying the hand measures in a vertical rather than horizontal direction. Two more students share other aspects that relate to Frederick's method of measuring. The above took place within the first two minutes of the lesson.

Ms. Opbroek says, "I would like to do a little bit more review but then also some more discussion on the activity we did. … So we created this angle, right, with our small objects and our large objects far away. What is the name of this piece of this drawing?" She draws on the board a large angle, relates it back to what the students were just talking about, and asks students to identify parts of an angle using mathematically correct vocabulary. Some students say it is a straw and others call out "ray." Shortly thereafter she asks, "What were we measuring? If you look at this angle here with the two rays, what do you suppose we were measuring in relation to this angle? And if you want to come to the board and draw you may." Kathy says, "The distance between the objects," goes to the

Figure 3. Alice demonstrates how Frederick uses his hands to measure between two objects.

board and draws what she is referring to. When prompted by Ms. Opbroek, all the students agree with Kathy's explanation.

However, Ms. Opbroek is not content to leave the conversation here. An angle can be defined as the amount of rotation between two rays or dynamically as the sweep of an arc or a turn through space (Lehrer, 2003). By introducing angles through the movement of shadows in the *Star Navigation* module, we are trying to instill this sense of angles as rotation. However, the idea of an angle as a measure of rotation is hard to internalize. It differs from the linear measures, which tend to be more of a centerpiece in elementary schools, because of the dissimilar nature of circles and lines. The more common linear measure, often measured using a ruler or a meter stick, is used when measuring length or distance. However, angles measuring rotation force the perception away from the beginning and end points used in distance and towards a center point (vertex) with starting and end points of a sweeping arm within a circle (Keiser, 1997).

Ms. Opbroek calls on a group of students that she remembers had different measurements for the same objects. She asks, "When you measured the distance between two objects, what was your measurement?" She continues to ask the other members of the group and summarizes her main point to the class: "same objects, different measurements." From the interviews with Ms. Opbroek, we found out that during this time she was intentionally setting up a classroom structure that invites contrary responses that students will have to reconcile through mathematical discourse. All the students were engaged up to this point as evidenced by kids using hand measures during student presentations, writing in their journals, watching the teacher and the board intently, and by the fact that they chose to sit in a circle.

At this point Ms. Opbroek begins a discussion on angles that may seem unconnected to the previous dialogue. She states, "We haven't really talked about it yet. We haven't talked about it at all. But all of you thought about your definitions of an angle. What is an angle, right?" She is referring to when students were asked to write in their journals their first definition of an angle at the end of Activity 2. She then shared with the class an anonymous summary of what many had written, such as "an angle is a degree." As Ms. Opbroek leads the discussion connecting Frederick's method of using hand measures to an understanding of angles, the following dialogue transpires:

Transcription: Minutes 9:30–10:27 Amy's Question	**Notes**
T: When you do this method are you measuring the angle?	Ms. Opbroek points to the drawing on the board of fingers used to measure between two far-away objects to refer to as the method.
Students: No.	Several students respond sporadically.
T: No?	Ms. Opbroek is patient and allows the students to think and respond. She does **not** give away the answer with her tone.
Collin: Yes	Confidently spoken.
T: Yes?	Again, she does **not** give away the answer with her tone.
T: If you say no, we're not measuring the degree of the angle. Then what are we measuring?	She pauses after asking this and then walks to the board.
Students: the distance … the distance in between two objects… that's what the angle is.	Several students respond in overlapping speech but somewhat quiet and reserved.
Amy: Isn't the distance the same thing as the angle?	As the teacher is at the board, but not speaking yet, Amy asks inquisitively. Ms. Opbroek stops what she is about to do, turns around and addresses this new question.
Students: No	Several quiet voices respond to Amy's question, some even whispering.

T: Those of you who said no raise your hand. Why? She said isn't the distance the same as the degrees of the angle and you said no.	Several students raise their hands as requested. She pauses before asking why. When no one begins to answer right away, she restates the question.

The room is rather quiet yet it is obvious that the students are still highly engaged. When asked yes or no, many respond in overlapping speech. Others are willing to answer differently from their friends. As students continue to answer directly to the teacher, others were quietly showing their agreement with smiles, body language, and whispers.

With Amy's question students are becoming aware of the conflict Ms. Opbroek has been trying to create. Ms. Opbroek is sensitive to the fact that students often confuse an angle measurement with linear distance and tend to focus on properties of angles that might lead to confusion. Research (Keiser, 1997) shows that students tend to focus on one of three aspects of angles: the vertex, the rays, or the interior. Each of these foci leads to difficulties in understanding the properties of angles.

<u>Focus on the vertex:</u> Students often define angles as the intersection of two rays, which focuses their understanding of angles on the vertex. Students with this focus often describe angles as being a "corner." However, this view of angles leads to some misunderstandings. When students think of an angle as a corner, they have difficulty understanding that angles can measure 180 degrees or more, or be exterior to a polygon.

<u>Focus on the rays:</u> Students often focus on the "sides" of an angle in their conceptualization of an angle. This conceptualization makes angle measurement difficult because students tend to use the length of the rays to determine angle size rather than their openness.

<u>Focus on interior of an angle:</u> Students can also think of angles as the amount of space between rays. Some students with this focus will be confused when it comes to measuring angles because the size of the angle will vary based on where along the rays students measure the arc. Like students who focus on the rays, students with an interior focus will often think of angles shown with longer rays as larger because the area of the interior space increases this way. Also, like those who focus on the vertex, students with a focus on the interior also have trouble visualizing angles greater than 180 degrees.

Although Amy's question becomes the main focus of the remainder of the discussion, the teacher remains in control of the classroom communication. During the next seven to nine minutes, students continue to share reasons why they think measuring the distance between two far-away objects is the same as measuring the degree of the angle or not. More students begin to explain their thinking to Ms. Opbroek, sometimes even confusing themselves. Students use protractors to point out the meaning of a degree, physical arguments such as, "it's like saying every degree is a mile," and connections back to the actual experiment of measuring with their hands. The class remains engaged in the conversation as seen by students mimicking hand measures while a classmate presents, students gazing at the presenter, and students moving around in their chairs to see better. Further evidence of engagement include other students speaking aloud during the presentations with affirming information, students reminding the presenter of the question to discuss, and some independently discussing their ideas with their neighbor. Students begin to say, "I agree with Amy" or "I want to change what I said before."

As the discussion continues, the teacher introduces new math vocabulary and models its proper use. Students continue to take turns sharing and mostly responding to the teacher. A new focus on defining the term "degree" has developed. Ms. Opbroek uses the situation to introduce new vocabulary: vertex. During this time, students continue to argue over what is actually being measured—is it the degrees or the distance—as seen in the following transcription.

Transcription: Minutes 14:35–15:38 Degrees vs. Distance	**Notes**
T: You are changing and going back to what Amy said.	Ms. Opbroek addresses Kathy.
Kathy: Because … I think it's because like the degrees is like the measurement between two rays …	Kathy turns to face Ms. Opbroek as she picks her words carefully to share her idea.
Amy: Thank you.	Amy interjects.
Kathy: … and um and also like the rays can be the objects.	Kathy continues with the idea that explains why she changed her mind.
T: Okay, so, when we do this activity like Frederick did, with our hands we are measuring the degrees …	Ms. Opbroek models with her hands as she speaks. Amy interjects the new idea as shared by Kathy of distance between two objects while Ms. Opbroek focuses on degrees.
Amy: the distance between the two rays	
T: … whether the objects are here or the objects are here or here we are still measuring degrees.	She points to the board at different locations on the ray to show the different meanings of the term "here."
T: Jared?	Ms. Opbroek notices that Jared wants to add to the conversation and calls on him with curiosity.

Jared: I don't think that's right because you are not measuring the degrees, you are measuring the distance between two objects.	Jared speaks directly to the teacher.
Mark: That's it! That's like saying every degree is a mile.	Mark speaks to the class.
Jared: Thank you.	Jared thanks Mark immediately.
Collin: Yeah, that's not true.	Collin directs his agreement with Jared towards the teacher.
Jared: I mean what you are saying is like we're only supposed to be measuring from the object to object. We are not supposed to be measuring degrees. Or … yeah.	Jared continues with his thought still addressing the teacher but allowing his speech to fall off as others tackle his idea.
Malcolm: Yes, we are. We are supposed to be measuring degrees.	Malcolm is sitting next to Jared and speaks directly to him.
Collin: Why wouldn't they just do degrees like inches and feet?	Collin changes his view from the class and Jared in particular to ask the question of the teacher directly.

During this minute of conversation many mathematical ideas are coming out, and students are becoming increasingly intrigued with the lack of agreement. Students are already sharing ideas that relate to the misconceptions of angles described above.

Ms. Opbroek now tries to get Mark to share because he has suggested ideas that shed light on the discussion. "Argue, Mark, defend me, we need it," states another student. With the help of other students, Ms. Opbroek talks Mark into sharing. There is a sense of victory when Mark finally goes to the board. "Yes, we got him," comes from several students. This type of student-to-student interaction and camaraderie between students aids in the persistence of the enthusiasm and dialogue around the mathematics.

The Shift

The following transcription highlights the students having an increasingly larger role and responsibility in convincing other students about their understanding of an angle.

Transcription: Minutes 17:28–19:00 The Conflict Continues	Notes
Mark: Okay. If you have two targets you can count the objects between them, but you can't count the degrees because it's like counting feet or miles. And you can't turn degrees into miles.	Mark demonstrates on the board as he speaks by drawing two circles for the targets or objects. He addresses the whole class.
Aimee: Or can you?	Spoken in a mysterious voice
Collin: You can't turn degrees into miles. Uh, can I? (asking permission to talk) That right there, that ain't a mile. If we want to change degree into mile, we have to go like this (demonstrates). It would have to go over what it could possibly go.	He begins to speak and then asks for permission by the teacher ("Can I?") before continuing. He demonstrates using his hands starting together and then spreading apart further and further.
Jake: That's 180 degrees. Jared: Not true, not true! Collin: Yes, true!	Lots of overlapping speech here. Students are calling out with lots of conviction and these three say the same thing over and over.
Jared: Not true. When we were measuring with the straws and the stacks and everything. When we were doing the little thingy. And we went like this. You could get on an open plane and you could have people measure it. And you go like this and you look at it. It could look like only two inches but it looks like two miles maybe.	Jared takes over the discussion and walks to the front as he speaks. The class gets quiet as he states his case. He models with his hands and acts out how to view from the vertex and how the measures might look different.
Collin: It depends on how you are measuring it though!	Collin speaks directly to Jared with determination.
Aimee: No it doesn't. Collin: Yeah it does. Kellie: I agree with Aimee. Collin: It depends on how you are measuring it though.	Others begin to add in their responses with lots of overlapping speech as Jared sits down. Even the teacher is carrying on a conversation with Malcolm only.
Jared: I decline my case.	Jared gives up as he is sitting down.
Thomas: I feel like I'm at a meeting.	This fifth-grader states his continued frustration.

Collin: I have one argument that I think everybody should understand.	Collin's exclamation towers over the low ramblings of the class as he raises his hand but is not addressed.
Amy (to observer): My question?	In the background Amy's question to the observer is heard.
Malcolm (to teacher): Well, it seems like that because it's the distance between two objects.	Then the conversation between Malcolm and Ms. Opbroek becomes louder.
Amy (to observer): Are we still on my question? Holy cow!	Again asking the observer and when she sees a head shake "yes" she is beaming with pride.
T: (to Malcolm) Say that again. (to the class) You guys, listen to what he says.	As the conversation between Malcolm and Ms. Opbroek comes to a close, she feels it's worth everyone hearing. The class falls silent.
Malcolm: Aren't we measuring the distance of the degrees?	Malcolm places his elbows on his knees and asks the whole class.
Students: no, yeah [overlapped speech]	Loud disagreement.
Malcolm: Yeah! Because here's the degrees and here's the distance between them.	Malcolm's "yeah" is louder than the others. He demonstrates with his hands by making a V to show degrees and then taps the air at imaginary points along a straight line farther away to show distance between them.
S: It's like the same thing.	A student out of the camera's view chimes in.
Jared: But we're measuring how far apart the objects are.	Jared argues with Malcolm directly.

During this segment, around minute 19, students begin responding to each other and the teacher supports this shift in communication. Students begin to team up with those who think similarly on the idea to convince the other school of thought to change. Ms. Opbroek no longer calls on students to share or adds in her comments after each student, but rather allows the students to carry the conversation. Students energetically run to the board, wanting to be the next one to share their proof and reasoning in favor or against the idea of the distance being the same as the degree of the angle. Students also take control of the conversation from their seats, and others turn their attention over to the one who is the loudest or seems the most convincing. We even hear from two of the fifth-grade students who have not shared much (Thomas and Jake). Their comments are less content oriented, but show that they are still involved in the conversation.

It is not entirely evident what initiated the shift in communication; however, teacher communication patterns, concepts introduced during the first 10 minutes, enthusiasm that is building among the students, and the growing tension between divergent thoughts could all have played a role. Ms. Opbroek had been setting up the class by moving them through the various concepts to head in this direction, providing them with the multiple related concepts with which she wanted them to struggle. She adapted the typical initiate, respond, evaluate (IRE) communicative pattern for a large group by dropping the evaluative component, relaxing the calling of specific students, and repeatedly providing an open invitation for sharing. This approach is becoming more documented in the literature on student argumentation and discourse (Forman, 2000). The mathematical conflict was becoming more evident to the students as the discussion continued. Students were able to relate to both sides of the conflict, discussing the confusion between comparing the distance between the two far-away objects with the hand measures from a specific perspective, thus approximating an angle measurement.

To better visualize the student discussion, refer to Figure 4. The straight line distances between the points AB and CD are different. The measure of the degrees formed by the rays is the same.

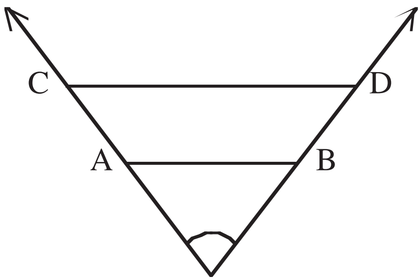

Figure 4. Drawing of the rays, angle, and points along the ray. The distance between objects is shown as a straight line.

Highlights from the remaining 1 hour and 10 minutes of the lesson follow. Minute 24:30–25:30: One student says excitedly to her neighbor, “I can actually prove why you can’t measure it with degrees.” Kathy addresses the issue by explaining to the class how we can measure with degrees because we can use a protractor for it. Minute 26: Thomas states that he is confused and lost. Supportive social norms allow him to admit this to the class and not feel uncomfortable. Prompted by the teacher, Thomas comes to the board to try using a protractor so the teacher can see if he can read degrees on the tool. Minute 31: Amy says that the distance and the degrees are the same. She proves it to herself drawing on the board but can't really describe it verbally to the class.

Minute 36: Alice summarizes the confusion saying, "OK. I say they are not the same distance apart because the objects inside are closer together and the objects outside are farther together and that's not the same distance. That is really different. Even though they are lined up it doesn't mean it's the same [moves her hands to show a growing V pattern]. If they were lined up like this [moves her hands to show parallel lines] they are the same, but they are like this [goes back to the V pattern] they get wider forever and ever and ever and they get wider and wider and they are not the same. I mean it's that easy. " Minute 37: Charles has only been in the class for three days but already he feels comfortable with sharing in front of everyone. Meanwhile three students (two fifth-graders and Amy) are still at the board working together to come up with their proof.

Minute 38: Teacher goes back to same hand measures but different question, asking, "When you measured the distance of your inside objects, you got the same measurements as the outside objects standing right where you were, right?" Alice: "No." Minute 38:44: Alice goes back to being confused since she didn't actually take both measurements when doing the experiment the day before. With bewilderment, Alice says, "When you hold your hands up you would get different hand measures. It would have to be less since they are closer together." Minute 42: Ms. Opbroek suggests taking a bathroom break and several students resist saying "no, we are not taking a break." Minute 43:30: Collin starts looking for physical objects to set up an experiment. Other students continue teaming up and discussing the topic. By minute 44:30 Ms. Opbroek insists on a break and students accept it, but run back excited and ready to continue.

Figure 5. Students line up to participate in the experiment Collin has set up.

Students come back from a bathroom break while Collin has been setting up his experiment. Ms. Opbroek helps to pull the class together to get them to focus on the task. She explains that this experiment was set up to answer a different question than the original one. The experiment will address if each person gets the same hand measure for both the nearby objects and the far-away objects. Ms. Opbroek is well aware of how this concept should help students understand the original question and is willing to allow the students to go in this direction.

This experiment helps address a few mathematical issues that keep arising, which stem from the ideas that are created from the previous activity. The math ideas include degrees versus distance, perspective versus real measurement, and the distance that the objects are apart are based on their location on rays that form an angle. These ideas are all related to each other and to what Frederick does in navigating.

As students continue gathering their data from the experiment, Thomas asks the quintessential math question.

Transcription: Minute 48 Thomas's Question	**Notes**
Thomas: Ms. Opbroek?	Thomas tries to get the teacher's attention a few times before she answers.
T: What?	Ms. Opbroek is in thought setting up the experiment as she acknowledges Thomas.
Thomas: Is Collin right or is he wrong?	He asks with a tone of tired frustration.
Students: [overlapped yelling] right, wrong, Collin is so wrong, he's right in one way and wrong in another. Collin: You guys aren't understanding what I'm saying.	Many students answer Thomas's question. Collin even responds to the whole class.
T: Thomas, what you'll learn in these units, what you'll learn in these units that we do is that I don't ever give you the answer.	Ms. Opbroek stops what she is doing, looks at Thomas, and answers him.
S: You have to figure it out for yourself.	Other students continue to answer Thomas.

The lesson now morphs into the students taking turns measuring with their hands and recording their two different measurements—for the close objects and the far-away objects. Engagement continues to remain high as all students either eagerly line up to be part of the experiment or watch their classmates take measurements until it is their turn. Further, students continue to work with each other and discuss their conjectures and proofs. As students share their data, Ms. Opbroek then leads them to try answering the question by looking for patterns.

Ms. Opbroek brings out the angle measurement that students have not quite addressed throughout the discussion. "The one thing we haven't done yet is we have not discussed what angle this is. I noticed you put a protractor down there. Collin will you please read the protractor for us."

Figure 6. Collin created a new tool by attaching a protractor to his straw angle tool.

While Collin is measuring, he reports 45 degrees. Jared screams out with a slight tone of exhaustion, "the degrees … stay … the same!" Another student replies emphatically, "yes they do!" Here, it was not observed whether students were able to connect this with what they were discussing earlier. Ms. Opbroek did not pick up on this sub-discussion either and continues to ask the students to report their hand measures. Then, as a whole class, they organize the table into groups—those with the same measurements and those with different measurements as suggested by Charles. Ms. Opbroek concludes the class by using this setup and the students' ideas and responses to help model how to design a project for the science fair, a requirement all the sixth-grade students must do within the next week. She writes on the board: "Is measuring the distance between objects the same as measuring the angle's degrees?" The class ends with journaling on this question.

Discussion

Often throughout the discussion, both the teacher and the students connected back to Frederick George and his knowledge. Frederick's knowledge is not everyday or cultural knowledge for anyone in the classroom but stems from a general theme of tapping into people's everyday surroundings while living in Alaska. As evident from literature, incorporating these elements into the mathematics curriculum can contribute to a more meaningful appreciation of mathematics (Gerdes, 1994). Further, the work of Gravemeijer (1994) on developing realistic mathematics education and Treffers' (1993) work on realistic mathematics education support this argument by demonstrating the need for realistic contexts to aid students in formulating mathematical ideas in meaningful ways (de Lange, 1992).

Students also relied on their own knowledge of measuring to pose conjectures and offer proof. Using the information in the MCC curriculum, Ms. Opbroek provided the framework of information for students to connect and fostered their thinking by never giving away the answer, being patient with the discussion, and allowing students the freedom to share ideas and offer a variety of proofs. Further, it is hoped that a thorough description of the mathematical content that the students were struggling to understand (angles, distance, etc.) and the class dynamics that allowed them to negotiate their understanding have been provided.

Several classroom processes can be identified that (a) encouraged math communication and argumentation among the students, (b) created an effective classroom use of problem solving, and (c) allowed for student-generated questions, conjectures, and proofs. From the very beginning of class, Ms. Opbroek provided invitations for students to share, to use other methods of explaining, to draw or use their hands to explain, and to disagree with others in the class. She spoke patiently and waited for students to think and respond instead of pushing them to go quickly. The mere fact of allowing the class to continue the discussion for 1 hour 30 minutes, when she originally thought it would only last about 10 minutes, shows a considerable amount of patience with the math and the students. The curriculum provided hands-on activities for students to physically relate to, and this allowed them to argue over results and generate their own conjectures, relationships, and proofs. With Ms. Opbroek's strong pedagogical and math background, the math notes provided in the module, and the discussions from the professional development workshop, she was able to see how many of the lines of inquiry that students initiated were related to their developing understanding of an angle. She was able to allow them to investigate these seemingly divergent topics while staying focused on the task at hand.

It may seem that the Caucasian students led the discussions, but if viewed closely, the three Athabaskan students who sat together (two sixth-graders and one fifth-grader) remained engaged during this discussion as well. Alice seemed to be the spokesperson for the group of three girls. Many times during the class, the girls would discuss among themselves their thoughts on the topic at hand. They would use a lot of body language to share with each other. The group joined in the activities as they unfolded. They seemed to enjoy the conversation despite their small amount of sharing as evidenced by their excitement, gestures, and working together. Alice continually related back to the hands-on experiences modeled by Frederick George in the module as a means to share evidence of her thinking.

How did Ms. Opbroek use and adapt cultural components of MCC to develop meaningful math understanding for students of another cultural group? On first analysis of this lesson, we thought that students related to the Yup'ik knowledge because they are Alaskan and many live with relatives who do outdoor activities such as hunting, fishing, and camping. However, cultural components beyond our scope and thinking became apparent when our Yup'ik consultants viewed the video. The following statements and summary are compiled from the transcription of the video analysis meeting.

The idea of ownership came up several times. "She didn't own anything—everybody had ownership and she allowed the kids to own the lesson too. She respected their input." This is an emerging idea in our research and refers to the kids not only having ownership of something created, but also of the flow of the lesson and the knowledge used within and developed from the lesson. The teacher facilitating the lesson by using guiding questions was even considered a way to allow students ownership of the lesson and the knowledge. "She started using words and questioning and then when the questioning started coming from the kids she didn't give answers, she let them answer. If she were telling all the answers, the lessons would still be hers."

Further ideas began developing, such as the comfort level of students and how that relates to their discussion, proofs, and willingness to agree and disagree. "All the students were comfortable with one another, saying I disagree." Other consultants pointed out that the students were not afraid to speak out and did not seem to be afraid of being wrong. "No one laughed at one another; they respected one another."

The discussion became extremely exciting when the Yup'ik consultants related Ms. Opbroek's actions to those of the elders. This idea began with a reference to the teacher stepping out of the conversation and allowing the students to carry it; she would intervene when needed and then would step out again. Evelyn Yanez said, "The teacher was like us; she taught like an elder. The way she moved away from the blackboard and became a listener, a participant is how we teach. When I was in the classroom, I would teach, then step back and become a participant. I would go down to their level and be a part of the learning." The phrase "like us" refers to being like a Yup'ik teacher. Further she said, "The men act like that in Togiak [a Yup'ik village in southwest Alaska], the way they discuss real life things … and then after all that discussion with the other men, then everyone will leave and my dad and his brother will be there still discussing.

But they still don't know what the answer is. But they think they know how to get there." One consultant made the connection between the elders' teachings, survival skills, and the physical aspect of the lesson. It was also brought out that the classroom setup (students sitting in a circle) and the quality of student discussions were similar to a qasgiq (men's community house) because each individual student acted as part of the whole group.

As the discussion continued a new concept began developing: that of harmony. Everyone was working towards a common goal, even if they seemed like they were arguing. This relates back to a phrase and idea that we have heard from many elders over the past decade: If we are of one mind then we can accomplish our goals. This example is a clear picture that despite the disagreement, the students and the teacher all wanted to understand the concept better and come to an agreement.

Despite what seemed like arguing, the Yup'ik consultants saw harmony in the video since everyone was aiming for the same end result of understanding. All students were comfortable with one another. They were not afraid to express their ideas and there was harmony, respect, and trust among the students and between the students and the teacher. This is essential for student learning. The teacher allowed students to have ownership of the lesson. Students were thinking and trying to prove their conjectures. The students continued their discussions, never giving up.

During interviews with Ms. Opbroek, she expressed many of the same ideas as the Yup'ik consultants. She said that the description of harmony given by the Yup'ik consultants is "exactly what it means to have a constructivist classroom: same end goal but each kid constructs their own path."

Conclusion

In summary, this case provides an example of a classroom where students are initiating mathematical propositions and explorations while the teacher facilitates. During the lesson, students take more and more ownership as they increase their rights and responsibility while they try to understand the concept of angles and measuring. This dynamic and evolving classroom environment appears to produce a cohesive and collaborative classroom environment. This, coupled with the teacher's mathematical knowledge and continuing long-term relationship with many of the students, are additional ingredients that reinforce the effectiveness of this lesson.

The teacher, student, and curricula factors are strongly intertwined throughout this lesson. Ms. Opbroek's class is a community where the teacher and students are co-constructors of knowledge. We see that this curriculum is a good fit pedagogically for Ms. Opbroek, and she has been able to adapt it meaningfully to make it even more powerful than as written. The module uses Frederick George's method as the foundation to discuss angles, but the manner in which Ms. Opbroek facilitated went far beyond the perceived impact of the lesson. This case is rare, and we believe it is due to the superb enactment of the curriculum by the teacher.

Among the hundreds of classroom videos recorded and viewed by MCC project staff, this one stands out because the students initiate new lines of inquiry with students responding to others mathematically, and using the Yup'ik culture as a basis for all of it.

What can be gleaned from this example that may be useful in other situations and settings? First, it's essential to note that all students can learn from the MCC curriculum, not just Alaska Native students. Furthermore, AN students can find something relevant to their own lives in the MCC curriculum to help them better relate to mathematics. Providing a purpose for mathematics and relating it to their experience and knowledge can provide students with information they can use to create conjectures and proofs and begin to problem-solve, leading to rigorous mathematics as further supported by research from Civil and Kahn (2001).

The professional development implications from this case are twofold: mathematical and pedagogical. Creating a broader and deeper understanding of mathematics for the teacher provides them with the necessary tools to make quick decisions and become better facilitators of a discussion like the one presented here. Furthermore, developing a pedagogical style that allows the teacher to feel comfortable with student inquiry and becoming a facilitator takes practice. Many teachers are not open to this type of pedagogy for fear of losing control, students not learning, students moving off on divergent thoughts or discussions taking too much time, just to name a few issues. These fears keep teachers from realizing classrooms that NCTM promotes to be healthy learning situations. This case shows that it can be done and, considering the test results, that it can be more effective than previous traditional methods. Further, a teacher professional development program that can help bring about an adaptive change will promote teachers being highly motivated to try out new strategies, increasing their willingness to implement new approaches in the classroom, and being open to self-reflection and self-learning (Begg, 1994; Clarke & Peter, 1993).

This case demonstrates that current rural and AI/AN trends can be reversed as evidenced by performance on student tests, the wonderful example of Ms. Opbroek as facilitator, a curriculum that relates to students' interests and knowledge, and a classroom environment that promotes math communication. Further, it provides examples of factors that other teachers, administrators, and teacher educators can employ in their own teaching and classes to create more effective math classrooms.

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Endnotes

[1]All student names have been changed.

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